BOĞAZİÇİ ÜNİVERSİTESİ

ISS/EC-2008-05

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RESEARCH PAPERS

College Admissions Game: Early Action or Early Decision?*

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In this paper, we study the long-played, yet until now unmodeled, college admissions game over early admissions plans using a many-to-one matching framework. We characterize the equilibrium strategies of each college involving its early quota out of its total capacity, and the set of admissible and deferred students within its applicant pool independently from the early admissions plans of the colleges in the market. Given these strategies, we show that for each college early action is a weakly dominant choice between early admissions plans.

Keywords: Many-to-one matching, early action, early decision, college admissions.

JEL Classification Numbers: C71, C78, D71.

*The authors respectively thank, for their hospitality, the University of Pennsylvania and M.I.T., where they visited during academic year 2006-2007. The authors would like to gratefully acknowledge the financial support for this study by Bogazici University Research Fund, Project No. 07HC101. The authors also thank Ozgur Yilmaz for helpful comments and discussions. The first draft of this paper was written while the second author was affiliated with Bogazici University. The second author acknowledges the support of Turkish Academy of Sciences, in the framework of Distinguished Young Scientist Award Program (TUBA-GEBIP). The usual disclaimer applies.

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1 Introduction

Substantial number of elite colleges today in the United States, are admitting students through two-tier admissions programs; early admissions and regular admissions.[2] Since its inception over fifty years ago, various early admissions plans; 'early action', 'early decision' and 'single-choice early action' are adopted by colleges to skim the best students in 'thinner' early admissions markets. Early decision requires that the student promises by a signed contract to enroll at the program if offered admission, while early action allows the student the chance to gain an admission decison in the fall with a non-binding commitment. Single-choice early action program allows the student to apply to no other early admissions program in the fall.[3]

Over the years, colleges have frequently changed their admissions plans. Avery et.al. (2003, p. 97) reports that in 1995-1996, Princeton, Stanford and Yale adopted early decision plan while Brown, Georgetown and Harvard had allowed single-choice early applications with a subsequent adoption of multiple-choice early action program in 1999-2000. Brown quickly switched to early decision program in 2001-2002, whereas Harvard, Stanford and Yale made a u-turn to single-choice early action program in 2003-2004.

Institutions that were late in changing their admissions rules accordingly quickly fell behind in the admissions game frequently played by the elite colleges. Avery et.al. (2003, p. 298) reports that after the last change by Stanford, Yale and Harvard in 2003-2004, "prominent Early Action colleges such as Georgetown, MIT, and the University of Chicago saw their early applications fall by 15 percent or more. ... Princeton, which did not change its Early Decision program at all, also suffered a decline of 23 percent in early

[2] According to the *NACAC 2005 Early College Application Directory*, 384 colleges and universities offer early application options for students (15 percent of all four-year colleges). Besides, nearly 70 percent of the 281 private institutions ranked by the *U.S. News College Guide* as the "Best National Universities" and the "Best Liberal Arts Colleges" in the United States offer an early admissions program (Avery et.al., 2003, pp. 1-2).

[3] Early action and early decision programs are usually on the early November deadline with a late December decision timetable. Regular decision offers a later Application Deadline of January 1, a Common Notification Date usually in early April, and a Common Reply Date of May 1.

applications."

Early admissions programs that have been used very widely and strategically in the last decade have also started to be openly criticized as complicating the admissions system, forcing students to act strategically, unleveling the playing field in college admissions and in particular, disfavoring low-income students.[4] Very recently, following Harvard's remarkable announcement in 2006 to eliminate early admission,[5] Princeton and Virginia also announced that they will terminate their early admissions programs.[6] However, some colleges and universities that offer early action programs argue that it is not clear how elimination of non-binding early admissions programs such as early action will result in admission of more low-income students, as recently voiced by Richard Levin, the President of Yale University.[7]

Currently, it seems like there exists no consensus over neither the justification nor the use of early admissions programs. What is agreeable may only be that in the near future early admissions programs will remain to be strategically used by some, if not the majority, of the colleges. Given this simple conjecture, we aim in this paper to unveil some strategic aspects of the long-played, yet until now unmodeled, college admissions game. To this end, we study college admissions problem, introduced by Gale and Shapley (1962) and reformulated by Roth (1985), in a model with early decision and (multiple-choice) early action and analyze the strategic issues faced by colleges in the selection of early admissions programs with a focus on the intertemporal allocation of the total capacities.

Our model considers many-to-one matching problems (markets) involving two periods: an early admissions period and a regular admissions period. There are two finite and disjoint sets of agents, colleges and students. Each college has a finite overall capacity that limits the number of students it can accept in the two periods, and each student can enroll to at most one

[4] Avery et.al. (2003, p. 137) finds that applying early provides an admission advantage which is approximately equivalent to 100 additional SAT-1 points. The study also confirms that the early applicants tend to come from affluent socioeconomic backgrounds, while students seeking for financial aid postpone their admission to the regular decision period.

[5] "Harvard to Eliminate Early Admission," *Harvard Gazette*, September 14, 2006.

[6] News@Princeton and UVA Today, November 25, 2006.

[7] See http://www.yale.edu/opa/president/statements/20060912.html.

school during the whole matching process. Each college has a preference relation over the set of groups of students which is responsive to its preference over the set of students and each student has a preference relation over the set of colleges and being unmatched. The capacities of colleges together with the preference profiles of colleges and students constitute a matching environment.

In the early admissions period, each college announces its early admissions plan, be early decision or (multiple-choice) early action, an early admissions quota out of its total capacity, an early list of collections of students who are considered as acceptable in the early admissions period. On the other side of the market, each student submits an early list of acceptable colleges in the early admissions period for each possible profile of admissions plans. A student's early list at any profile of plans can contain at most one college that offers early decision. It is also assumed that keeping the early admissions plans of all colleges except for the plan of a particular college constant, the set of colleges that a student will apply when this college offers an early action plan includes the set of colleges that he or she would apply when this college instead offered an early decision plan.[8] Finally, given the early list of students, hence the early applicant pool of each college, the pre-matching actions of each college gives the list of the set of rejected, outright deffered, and admissible students in the early admissions period. The admissions plans and quotas of colleges, the early lists of colleges and students together with the pre-matching actions of colleges in the early admissions period define an early decision market.

An allocation in the early admissions period is a many-to-one *early matching* where no college is assigned more students than its early decision quota and no student is assigned more than one college. We assume that any college defers in the early admissions period, in addition to its outright deffered aplicants, any admissible student with whom it is not matched. Given a pre-matching early admissions market and a matching in early admissions, *post-matching early admissions status* of student determines the status of

[8]This assumption is statistically in line with the report of Avery et.al. (2003, p. 297) that when Stanford and Yale switched from early decision to single-choice early action, early applications they received increased by 70 percent and 56 percent, respectively.

a student at each college as 'not applied', 'rejected', 'ex-post deferred' or 'accepted'.

Given a matching realized in the early admissions period, a regular admissions market consists of regular lists of colleges which is a collection of acceptable students by colleges in regular admissions and regular lists of students which is a list of acceptable colleges by a student under his or her post-matching early admissions status. No student ever applies, in the regular admissions period, to a college that rejected himself or herself in early admissions. If a student is accepted to an early decision college in the early admissions period then he or she does not apply to any colleges in the regular admissions period. In addition, a student always applies in regular admissions to an early action college that gave ex-post defferal or acceptance as well as to an early decision college that gave ex-post deferral, provided there exists no other college that gave early acceptance to this student and is strictly preferred by the student. Moreover, we assume that the set of colleges a student applies in regular admissions when he or she was ex-post deffered by a college in early admissions involves all the colleges that he or she would apply when he or she was accepted early by that college plus a subset of the acceptable, but less preferred, colleges.

An allocation in the regular admissions period is a many-to-one *regular matching* where no college is assigned more students than its overall quota and no student is assigned more than one college. Notice that regular matching function preserves the early matchings of each college offering early decision plan.

A matching in the early admissions period is stable if no student prefers remaining unassigned to his or her assignment, no college prefers having a student slot vacant rather than filling it with one of its assignments, and there exists no unmatched college-student pair such that the college prefers the student to one of its assignments or keeping a vacant slot (if any) and the student prefers the college to his or her assignment. The last condition applies only to students applying colleges offering early action plans. Stability of a regular matching in regular admissions is defined similarly.

An early decision matching rule selects a matching for every early decision market and is stable if it selects a stable matching for every early decision market.

A regular decision matching rule selects a matching for every regular decision market, given any matching in any early decision market. A regular decision matching rule is stable at an early decision matching rule if it selects a stable matching for every regular decision market, given any realization of the early decision matching rule in any early decision market induced by the associated regular decision market.

An early decision rule and a regular decision rule as an ordered pair form a *matching system*. A matching system is stable if it involves a stable early decision rule at which the regular matching rule in the system is also stable.

Having just described a two-period college admissions market, we are now ready to consider an admissions game with observable actions, where the preferences of colleges and students and the total capacity of each college are common knowledge. The game consists of three consecutive stages: In the first stage, colleges simultaneously choose one of the two early admissions plans, namely early decision and (multiple-choice) early action, to be implemented in the second stage of the game. Once a profile of plans is determined, it becomes common knowledge in the rest of the game. In the second stage, the early admissions market opens. Observing the profile of early admissions plans and the corresponding preferences of students, each college simultaneously chooses its early preference list, its early quota, and the pre-matching partition of its applicant pool. Given the associated early admissions market, the early matching rule (run by a central clearinghouse) specifies for each college the list of students to accept early within its early admissions quota. Consequently, the post-matching summary of the early admissions market becomes common knowledge. In stage 3, the regular admissions market opens. Given the commitments of colleges in the early admissions period, vacant slots of colleges are filled according to the regular matching rule.

Since the third stage of the game involves no strategies to be played, we proceed from stage 2 backwards to solve for a subgame perfect equilibrium of the game. We first show that given any profile of early admissions plans, in stage 2 of the described game each college finds it a weakly dominant strategy to report its early quota being equal to its total capacity, its early preference list as the restriction of its regular preference ordering on any collection of acceptable students that involve the top acceptable students within its capacity size, the set of rejected students consisting of all students that are not acceptable with respect to its regular preference ordering, the set of outright deferred students to be consisting of all the nonrejected student applicants ranking outside its capacity size with respect to its regular preference ordering, and the set of early admissible students as simply the rest of the early applicants (Theorem 1).

Our second result shows that given the described equilibrium strategy in the second stage of the game, each college finds it a weakly dominant strategy to choose early action program in the first stage (Theorem 2).

Two related papers to ours are definitely by Konishi and Ünver (2006) and Mumcu and Saglam (2007), deserving a lengthy discussion. Konishi and Ünver (2006) model a game of capacity manipulation for hospital-intern markets with a single decision period. They show that there may not be a pure strategy equilibrium under hospital-optimal and intern-optimal stable rules, and whenever a pure strategy equilibrium exists, every hospital weakly prefers this equilibrium outcome to the outcome of any larger capacity profile. Konishi and Ünver (2006) consider two restrictions on preferences to ensure the existence of a pure strategy equilibrium. Under the first restriction, hospitals always prefer a larger set of acceptable interns to a smaller set. Then, reporting the number of assigned interns is an equilibrium strategy if the matching rule is hospital-optimal and reporting the actual capacity is a weakly dominant strategy if the matching rule is intern-optimal. The second restriction requires common preferences of one group of agents (hospitals or interns) over the other group and guarantees that reporting the true capacity is a weakly dominant strategy for each hospital.

Mumcu and Saglam (2007) study early admissions problem; however colleges, unlike in our model, do not play a game over early admissions programs. Their model, which borrows from the one-period game in Konishi and Ünver (2006), considers a two-period admissions market where all colleges have an early decision program and play a noncooperative game of the intertemporal allocation of the total capacity. Under college-optimal and student-optimal matching systems, they show that there may not be a pure strategy equilibrium. Therefore, Mumcu and Saglam (2007) restrict preferences to ensure the existence of pure strategy equilibria. They prove that when either colleges or students have common preferences over the other set of agents, 'terminating early decision program' becomes a weakly dominant strategy for each college if every student, choosing to act early, applies to his or her top choice college irrespective of the early decision quotas of colleges. We should immediately notice that the partial result of Mumcu and Saglam (2007) to eliminate early decision is checked by our second theorem to be robust with respect to the presence of colleges in the market offering early action programs and with respect to the less restrictive preferences of colleges and students. However, our result clearly conveys more: a two-period college admissions involving early action, as the additional plan we consider in our paper, is weakly dominating, for colleges, the one-period regular admissions, as well.

The organization of the paper is as follows: Section 2 introduces the model. Section 3 presents our results and Section 4 concludes.

2 Model

2.1 Basic Structures

2.1.1 Matching Environment

A matching environment is denoted by the list (C, S, q, R) that involves the following fixed components:

Market Participants: The first two components of a matching environment are non-empty, finite and disjoint sets of colleges $C = c_1, c_2, ..., c_m$ and students $S = s_1, s_2, ..., s_n$.

Total Capacities of Colleges: The third component is a vector of positive natural numbers $q = (q_{c_1}, ..., q_{c_m})$, where q_c is the total capacity of college c.

Preferences: The last component of a matching environment is a list of preference relations $R = (R_{c_1}, ..., R_{c_m}, R_{s_1}, ..., R_{s_n})$ where R_c is the preference relation of college c and R_s is the preference relation of student s.

For any $c \in C$, R_c is a binary preference relation that is a linear order on $\Sigma_c = 2^S$. Similarly, for any $s \in S$, R_s is a binary relation that is a linear order on $\Sigma_s = \{\{c_1\}, \{c_2\}, ...\{c_m\}, \emptyset\}$. The element $\emptyset$ is interpreted by both colleges and students as the prospect of being unassigned. Let $\mathcal{R}_c$ and $\mathcal{R}_s$ respectively denote the class of all preference relations for college $c \in C$ and for student $s \in S$. Define also $\mathcal{R} = \times_{k \in C \cup S} \mathcal{R}_k$. Let P_k denote the strict preference relation associated with the preference relation R_k for agent $k \in C \cup S$. We say that an element $y \in \Sigma_x$ is acceptable to agent $x \in C \cup S$ if $y \, P_x \, \{\emptyset\}$.

The preference relation R_c of college $c \in C$ is said to be responsive (Roth, 1985) whenever for all $S' \subset S$ it is true that

i) for all $s \in S \backslash S'$, $S' \cup \{s\} \, P_c \, S'$ if and only if $\{s\} P_c \, \emptyset$,

ii) for all $s, s' \in S \backslash S'$, $S' \cup \{s\} \, P_c \, S' \cup \{s'\}$ if and only if $\{s\} P_c \{s'\}$.

(Notice that preferences of students over the individual colleges are trivially responsive.)

Let $\mathcal{F}$ denote the set of all matching environments. For the rest of the model, we fix the matching environment at $F = (C, S, q, R)$.

2.1.2 Early Admissions Market

An early admissions market is a list $(\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$ with its components defined as follows:

College Admissions Plans: The first component is a vector of college admissions plans $\alpha = (\alpha_{c_1}, ..., \alpha_{c_m})$, where for all c we have $\alpha_c \in \{a, d\}$, with a and d respectively denoting '(multiple-choice) early action plan' and 'early decision plan' to be implemented in the early admissions period.

Early Quotas of Colleges: The second component is a vector of nonnegative natural numbers $q_C^E(\alpha) = (q_{c_1}^E(\alpha), ..., q_{c_m}^E(\alpha))$, where $q_c^E(\alpha) \in \{0, 1, ..., q_c\}$ is the quota of college c in the early admissions period under the early admissions profile α. Define $\mathcal{Q}_c^E(q_c) = \{0, 1, ..., q_c\}$ for all $c \in C$ and $\mathcal{Q}^E(q) =$

$\times_{c\in C}\, \mathcal{Q}_c^E(q_c)$.

Early Lists of Colleges: The third component is a vector of lists $\Sigma_C^E(\alpha) = (\Sigma_{c_1}^E(\alpha), ..., \Sigma_{c_m}^E(\alpha))$ where $\Sigma_c^E(\alpha)$ is the ordered list of collections of students, each of which is considered as acceptable by college c in the early admissions period under the profile of early admissions plans α. Thus, we will assume that $\Sigma_c^E(\alpha) \subseteq \{T \in \Sigma_c : T\, P_c\, \emptyset\}$ for all c and for all $\alpha \in \{a, d\}^m$. We will simply call $\Sigma_C^E(\alpha)$ as 'early lists of colleges' under the plan α.

Early Lists of Students: The fourth component is a vector of lists $\Sigma_S^E(\alpha) = (\Sigma_{s_1}^E(\alpha), ..., \Sigma_{s_n}^E(\alpha))$ where $\Sigma_s^E(\alpha)$ is the ordered list of colleges each of which is considered as acceptable by student s in the early admissions period under the profile of early admissions plans α. We will simply call $\Sigma_S^E(\alpha)$ as 'early lists of students' under the plan α.

For any $\alpha \in \{a, d\}^m$, we will assume that $\Sigma_s^E(\alpha) \subseteq \{c \in \Sigma_s : \{c\}\, P_s\, \emptyset\}$ such that $|\{c \in \Sigma_s^E : \alpha_c = d\}| \leq 1$ for all s. Moreover, for all $c \in C$, for all $\alpha_{-c} \in \{a, d\}^{m-1}$, and for all $s \in S$, we assume $\Sigma_s^E((a, \alpha_{-c})) \supseteq \Sigma_s^E((d, \alpha_{-c}))$, i.e., keeping the early admissions plans of all colleges except for the plan of college c constant, the set of colleges that student s would apply when college c offers an early action plan includes the set of colleges that he or she would apply when college c offers an early decision plan.[9]

Pre-Matching Actions of Colleges in the Early Admissions Period: The last component of an early admissions market summarizes the pre-matching actions of colleges. Given a list $\Sigma_S^E(\alpha)$, we define the early applicant pool for any college c by $\Phi_c^E(\alpha) = \{s \in S : c \in \Sigma_s^E(\alpha)\}$. Let $J_c^E(\alpha) \subseteq \Phi_c^E(\alpha)$ denote the set of students rejected by college c in early admissions. For any college c, let the set of outright deferred students in the early admissions be denoted by $D_c^E(\alpha) \subseteq \Phi_c^E(\alpha) \backslash J_c^E(\alpha)$. Then the resulting set of early admissible students for college c is denoted by $A_c^E(\alpha) = \Phi_c^E(\alpha) \backslash (D_c^E(\alpha) \cup J_c^E(\alpha))$. Define a partition $\Pi_c^E(\alpha) = \{J_c^E(\alpha), D_c^E(\alpha), A_c^E(\alpha)\}$ as the

[9]This assumption is consistent with the fact that any college offering an early decision plan cannot receive early application from any student who applies to another early decision plan, since students cannot apply to more than one early decision plan.

summary of the pre-matching actions of college c. Let $J_C^E(\alpha) = (J_c^E(\alpha))_{c\in C}$, $D_C^E(\alpha) = (D_c^E(\alpha))_{c\in C}$, $A_C^E(\alpha) = (A_c^E(\alpha))_{c\in C}$, and $\Pi_C^E(\alpha) = (\Pi_c^E(\alpha))_{c\in C}$.

Let $\mathcal{Z}^E$ denote the set of all possible early admissions markets. Denote by $\mathcal{Z}_{\alpha,c}^E$ the set of all possible lists $(q_c^E(\alpha), \Sigma_c^E(\alpha), \Pi_c^E(\alpha))$. Define $\mathcal{Z}_{\alpha,C}^E(i) = \times_{c\in C}\mathcal{Z}_{\alpha,c}^E(i)$ for $i \in \{1, 2, 3\}$, $\mathcal{Z}_{\alpha,C}^E = (\mathcal{Z}_{\alpha,C}^E(1), \mathcal{Z}_{\alpha,C}^E(2), \mathcal{Z}_{\alpha,C}^E(3))$, and $\mathcal{Z}_{\alpha,-c}^E = \mathcal{Z}_{\alpha,C\setminus\{c\}}^E$. Also define the completion operator $\Gamma_\alpha : \mathcal{Z}_{\alpha,C}^E \to \mathcal{Z}^E$, which will be useful in Section 2.2., such that $\Gamma_\alpha(Z) = (\alpha, Z(1), Z(2), \Sigma_S^E(\alpha), Z(3))$ for any $Z \in \mathcal{Z}_{\alpha,C}^E$, where $Z(i)$ denotes the ith component of Z.

2.1.3 Early Admissions Matching Problem

Early Matchings: Given an early admissions market $(\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, a matching μ^E in the early admissions period is a function from the set $C \cup S$ into $2^{C\cup S}$ such that:

i) $|\mu^E(s)| \leq 1$ and $\mu^E(s) \subseteq \Sigma_s^E(\alpha)$ for all $s \in S$;

ii) $|\mu^E(c)| \leq q_c^E(\alpha)$ and $\mu^E(c) \subseteq \Sigma_c^E(\alpha)$ for all $c \in C$;

iii) for all $(c, s) \in C \times S$, $\mu^E(s) = \{c\}$ if and only if $s \in \mu^E(c)$.

We denote the set of all matchings for a given early admissions market $Z^E \in \mathcal{Z}^E$ by $\mathcal{M}^E(Z^E)$. Let $\mathcal{M}^E = \cup_{Z^E\in\mathcal{Z}^E}\mathcal{M}^E(Z^E)$.

We say that s prefers matching μ_1^E to matching μ_2^E if and only if it prefers $\mu_1^E(s)$ to $\mu_2^E(s)$ under the preference relation R_s. We do the same for each college.

Admissible Early Choices of Colleges: Given an early admissions market $Z^E = (\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, the admissible choice of a college c from a group of students $T \subseteq S$ in the early admissions market is defined as

$$Ch_c^E(T, Z^E) = \{T' \subseteq T \cap A_c^E(\alpha) : |T'| \leq q_c^E(\alpha), \quad T' \, R_c \, T''$$

$$\text{for all } T'' \subseteq T \text{ such that } |T''| \leq q_c^E(\alpha)\}.$$

Blocking Early Matchings: Given an early admissions market $Z^E = (\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, a matching $\mu^E \in \mathcal{M}^E(Z^E)$ is blocked by college c if $\mu^E(c) \neq Ch_c^E(\mu^E(c), Z^E)$.[10] A matching μ^E is blocked by a college-student pair (c, s) under early action if $c \in \Sigma_s^E(a)$, $s \in \Sigma_c^E(a)$, $\{c\}\, P_s\, \mu^E(s)$, and $\mu^E(c) \neq Ch_c^E(\mu^E(c) \cup \{s\}, Z^E)$.[11]

Stability of Early Matchings: A matching μ^E is stable if it is not blocked by a student, a college, or a college-student pair. We denote by $\mathcal{S}^E(Z^E)$, the set of stable matchings for the early admissions market $Z^E \in \mathcal{Z}^E$. In this set, there exists a matching, $\mu_C^E(Z^E)$, called the college-optimal stable matching in the early admissions period such that

$$\mu_C^E(Z^E)(c)\, R_c\, \mu^E(c)$$

for all $c \in C$ and for all $\mu^E \in \mathcal{S}^E(Z^E)$.

Analogously, there is a student-optimal stable matching in the early admissions period, $\mu_S^E(Z^E)$, that every student likes as well as any other stable matching.

Early Matching Rules: A matching rule in the early admissions period is a function $\varphi^E : \mathcal{Z}^E \to \mathcal{M}^E$ such that for all $Z^E \in \mathcal{Z}^E$, we have $\varphi^E(Z^E) \in \mathcal{M}^E(Z^E)$. Let $\bar{\varphi}^E$ denote the set of all early matching rules.

Stability of Early Matching Rules: An early matching rule $\varphi^E \in \bar{\varphi}^E$ is stable if $\varphi^E(Z^E) \in \mathcal{S}^E(Z^E)$ for all $Z^E \in \mathcal{Z}^E$.

Ex-post Deferred Students in the Early Admissions Period: We assume that any college $c \in C$ defers in the early admissions period, in addition to its outright deferred applicants, any admissible student with whom it is not matched. That is, given an early admissions market $(\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, college c late defers all students in $A_c^E(\alpha) \backslash \mu^E(c)$. Let $\tilde{D}_c^E(\alpha) =$

[10] We do not need to define and check the usual individual blocking of students since by definition $\mu^E(s)\, R_s\, \{\emptyset\}$ for all $s \in S$.

[11] Note that under early decision, the student applies to a single college, hence pairwise blockings are not possible.

$D_c^E(\alpha) \cup A_c^E(\alpha) \backslash \mu^E(c)$ denote the set of ex-post deferred students after the early admissions market is closed. Define $\tilde{D}_C^E(\alpha) = (\tilde{D}_c^E(\alpha))_{c \in C}$. Also define $\tilde{\Pi}_c^E(\alpha) = \{J_c^E(\alpha),\ \tilde{D}_c^E(\alpha), \mu^E(c)\}$ and $\tilde{\Pi}_C^E(\alpha) = (\tilde{\Pi}_c^E(\alpha))_{c \in C}$.

Post-Matching Summary of Early Admissions Market: Given a pre-matching early admissions market $Z^E \in \mathcal{Z}^E$ and a matching function $\mu^E \in \mathcal{M}^E(Z^E)$, let the vector $Z_{\mu^E}^E$ denote the corresponding post-matching early admissions market such that $Z_{\mu^E}^E(i) = Z^E(i)$ for $i \neq 5$ and $Z_{\mu^E}^E(5) = \tilde{\Pi}_C^E(\alpha)$, where $Z_{\mu^E}^E(i)$ and $Z^E(i)$ respectively denote the ith component of the vectors $Z_{\mu^E}^E$ and Z^E.

Post-Matching Early Admissions Status of Students: Given an early admissions market $Z^E = (\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, an early matching $\mu^E \in \mathcal{M}^E(Z^E)$, define the status of student s at college c with respect to the post-matching summary $Z_{\mu^E}^E$ of Z^E as

$$x_{s,c}^E(Z_{\mu^E}^E) = \begin{cases} 0 \text{ (not applied)} & \text{if } s \notin \tilde{\Pi}_C^E(\alpha), \\ 1 \text{ (rejected)} & \text{if } s \in J_C^E(\alpha), \\ 2 \text{ (ex-post deferred)} & \text{if } s \in \tilde{D}_C^E(\alpha), \\ 3 \text{ (accepted)} & \text{if } s \in \mu^E(c). \end{cases}$$

Let $x_s^E(Z_{\mu^E}^E) = (x_{s,c}^E(Z_{\mu^E}^E))_{c \in C} = (x_{s,c}^E(Z_{\mu^E}^E), x_{s,-c}^E(Z_{\mu^E}^E))$ for any $c \in C$ and $x_S^E(Z_{\mu^E}^E) = (x_s^E(Z_{\mu^E}^E))_{s \in S} = (x_s^E(Z_{\mu^E}^E), x_{-s}^E(Z_{\mu^E}^E))$ for any $s \in S$.

2.1.4 Regular Admissions Market

Given an early admissions market $Z^E = (\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$ and an early matching $\mu^E \in \mathcal{M}^E(Z^E)$, a regular admissions market is a list $Z^R(Z^E) = (\Sigma_C^R(Z_{\mu^E}^E), \Sigma_S^R(x_S^E(Z_{\mu^E}^E)), \Pi_C^R(\Sigma_S^R(x_S^E(Z_{\mu^E}^E)))$ with its components defined as follows:

Regular Lists of Colleges: The first component of a regular matching market is a vector of lists $\Sigma_C^R(Z_{\mu^E}^E) = (\Sigma_c^R(Z_{\mu^E}^E))_{c \in C}$, where $\Sigma_c^R(Z_{\mu^E}^E)$ is the ordered list of collection of students each of which is considered as acceptable by college c in the regular admissions period. We assume that $\Sigma_c^E(\alpha) \subseteq \Sigma_c^R(Z_{\mu^E}^E) \subseteq$

Σ_c for all c. We will simply call $\Sigma_C^R(Z_{\mu^E}^E)$ as 'regular lists of colleges'.

Regular Lists of Students: The second component of a regular matching market is a vector of lists $\Sigma_S^R(x_S^E(Z_{\mu^E}^E)) = (\Sigma_s^R(x_S^E(Z_{\mu^E}^E)))_{s\in S}$, where $\Sigma_s^R(x_S^E(Z_{\mu^E}^E))$ is the ordered list of colleges each of which is considered as acceptable by student s in the regular admissions period under the post-matching early admissions status $x_S^E(Z_{\mu^E}^E)$ of students. We will simply call $\Sigma_S^R(x_S^E(Z_{\mu^E}^E))$ as 'regular lists of students'.

We assume that $\Sigma_S^R(x_S^E(Z_{\mu^E}^E)) \subseteq \{c \in \Sigma_s : \{c\}\, P_s\, \emptyset\}$. However, for any s and any c such that $x_{s,c}^E(Z_{\mu^E}^E) = 1$, we have $c \notin \Sigma_s^R(x_S^E(Z_{\mu^E}^E))$, i.e., no student ever applies in the regular admissions period to a college that rejected himself or herself in early admissions. For any s and any c such that $\alpha_c = d$ and $x_{s,c}^E(Z_{\mu^E}^E) = 3$, we have $\Sigma_s^R(x_S^E(Z_{\mu^E}^E)) = \emptyset$, i.e., a student who has been accepted early to an early decision program does not make any regular applications. In addition, for any s and any c such that either $x_{s,c}^E(Z_{\mu^E}^E) = 3$ and $\alpha_c = a$ or $x_{s,c}^E(Z_{\mu^E}^E) = 2$, we have $c \in \Sigma_s^R(x_S^E(Z_{\mu^E}^E))$ if and only if there exists no c' such that $x_{s,c'}^E(Z_{\mu^E}^E) = 3$ and $\{c'\}\, P_s\, \{c\}$, i.e., a student always applies in regular admissions to an early action college that gave ex-post deferral or acceptance as well as to an early decision college that gave ex-post deferral, provided there exists no other college that gave early acceptance to this student and is strictly preferred by the student. Moreover, we assume that the set of colleges a student s applies in regular admissions when he or she was ex-post deferred by a college c in early admissions involves all the colleges that he or she would apply when he or she was accepted early by college c plus a subset of the colleges that are less preferred than c and more preferred than $\emptyset$, keeping constant the early status of this student at all other colleges. This last assumption will be called *precautiosness of students.*

Pre-Matching Actions of Colleges in the Regular Admissions Period: Given the regular applicant pool $\Phi_c^R(Z_{\mu^E}^E) = \{s \in S : c \in \Sigma_s^R(x_S^E(Z_{\mu^E}^E))\}$ of college c, we simply assume that in the regular admissions period the set of outright rejected students by college c is $J_c^R(Z_{\mu^E}^E) = \Phi_c^R(Z_{\mu^E}^E) \backslash \Sigma_c^R(Z_{\mu^E}^E)$ for each college c. We then denote by $A_c^R(Z_{\mu^E}^E) = \Phi_c^R(Z_{\mu^E}^E) \cap \Sigma_c^R(Z_{\mu^E}^E)$ the set of admissible students for college c in the regular admissions period. Let

$\Pi_c^R(Z_{\mu^E}^E) = \{J_c^R(Z_{\mu^E}^E), A_c^R(Z_{\mu^E}^E)\}$. Also define $J_C^R(Z_{\mu^E}^E) = (J_c^R(Z_{\mu^E}^E))_{c\in C}$, $A_C^R(Z_{\mu^E}^E) = (A_c^R(Z_{\mu^E}^E))_{c\in C}$, and $\Pi_C^R(Z_{\mu^E}^E) = (\Pi_c^R(Z_{\mu^E}^E))_{c\in C}$.

Let $\mathcal{Z}^R(Z_{\mu^E}^E)$ denote the set of all regular admissions markets given the post-matching summary $Z_{\mu^E}^E$ of the early admissions market Z^E. Define $\mathcal{Z}^R = \cup_{Z^E \in \mathcal{Z}^E} \cup_{\mu^E \in \mathcal{M}^E(Z^E)} \mathcal{Z}^R(Z_{\mu^E}^E)$.

2.1.5 Regular Admissions Matching Problem

Regular Matchings: Given a regular admissions market $(\Sigma_C^R(Z_{\mu^E}^E), \Sigma_S^R(x_S^E(Z_{\mu^E}^E)), \Pi_C^R(Z_{\mu^E}^E))$ defined at $Z_{\mu^E}^E$, we define a matching μ^R in the regular admissions period as a function from the set $C \cup S$ into $2^{C\cup S}$ such that:

i) $|\mu^R(s)| \leq 1$ and $\mu^R(s) \subseteq \Sigma_s^R(x_S^E(Z_{\mu^E}^E))$ for all $s \in S$, and $\mu^R(s) = \mu^E(s)$ for any $s \in S$ such that $\alpha_{\mu^E(s)} = d$;

ii) $|\mu^R(c)| \leq q_c$ and $\mu^R(c) \subseteq \Sigma_c^R(Z_{\mu^E}^E)$ for all $c \in C$, and $\mu^R(c) \supseteq \mu^E(c)$ for any $c \in C$ such that $\alpha_c = d$;

iii) for all $(c, s) \in C \times S$, $\mu^R(s) = \{c\}$ if and only if $s \in \mu^R(c)$.

We notice that the function μ^R preserves the early admissions matchings under μ^E of each college offering an early decision plan. Here, we denote the set of all matchings in the regular admissions period for a given regular admissions matching problem $Z^R(Z_{\mu^E}^E) \in \mathcal{Z}^R(Z_{\mu^E}^E)$ by $\mathcal{M}^R(Z^R(Z_{\mu^E}^E))$ and the set of all such matchings in the regular admissions period by $\mathcal{M}^R(Z_{\mu^E}^E)$. We also define $\mathcal{M}^R = \cup_{Z^E \in \mathcal{Z}^E} \cup_{\mu^E \in \mathcal{M}^E(Z^E)} \mathcal{M}^R(Z_{\mu^E}^E)$.

We say that s prefers matching μ_1^R to matching μ_2^R in the regular admissions period if and only if s prefers $\mu_1^R(s)$ to $\mu_2^R(s)$ under the preference relation R_s. We do the same for each college.

Admissible Regular Choices of Colleges: Given a post-matching early admissions market $Z_{\mu^E}^E$ and a regular admissions market $Z^R(Z_{\mu^E}^E) = (\Sigma_C^R(Z_{\mu^E}^E), \Sigma_S^R(x_S^E(Z_{\mu^E}^E)), \Pi_C^R(Z_{\mu^E}^E))$, the admissible regular choice of a college c from a group of students T available for assignment satisfying $T \subseteq S \backslash \mu^E(c)$, is defined as

$$Ch_c^R(T, Z^R(Z_{\mu^E}^E)) = \{T' \subseteq T \cap A_c^R(Z_{\mu^E}^E) : |T'| \leq q_c - |\mu^E(c)|,$$

$$T' \cup \mu^E(c)\, R_c\, T'' \cup \mu^E(c) \text{ for all } T'' \subseteq T$$
$$\text{such that } |T''| \leq q_c - |\mu^E(c)|\}.$$

Note that irrespective of the type of its early admissions plan, the admissible choice of every college in the regular admissions period always selects from a set that excludes the students to which the college early committed itself.

Blocking Regular Matchings: Given a post-matching early admissions market $Z^E_{\mu^E}$ and a regular admissions market $Z^R(Z^E_{\mu^E}) = (\Sigma^R_C(Z^E_{\mu^E}), \Sigma^R_S(x^E_S(Z^E_{\mu^E})), \Pi^R_C(Z^E_{\mu^E}))$, a regular admissions matching μ^R preserves the matches between students and colleges offering early decision plan, i.e., $\mu^R(s) = \mu^E(s)$ for any s such that $\mu^E(s) \in C$ and $\alpha_{\mu^E(s)} = d$. In fact, only those students who are in the early admissions period assigned to a college offering early action plan can block a regular admissions matching.[12] That is, a matching μ^R is blocked in the regular admissions period by any student s such that $\mu^E(s)\, P_s\, \mu^R(s)$ only if $\alpha_{\mu^E(s)} = a$. A matching μ^R is blocked by college c in the regular admissions period if $\mu^R(c)\backslash\mu^E(c) \neq Ch^R_c(\mu^R(c)\backslash\mu^E(c), Z^R(Z^E_{\mu^E}))$. A matching μ^R is blocked in the regular admissions period by a college-student pair (c, s) satisfying either $\mu^E(s) = \emptyset$ or else $\alpha_{\mu^E(s)} = a$ if $c \in \Sigma^R_s(x^E_S(Z^E_{\mu^E}))$, $s \in \Sigma^R_c(Z^E_{\mu^E})$, $\{c\}\, P_s\, \mu^R(s)$, and $\mu^R(c)\backslash\mu^E(c) \neq Ch^R_c(\{s\} \cup \mu^R(c)\backslash\mu^E(c), Z^R(Z^E_{\mu^E}))$.

Stability of Regular Matchings: A matching μ^R is stable if it is not blocked by a student, a college, or a college-student pair. We denote by $\mathcal{S}^R(Z^R(Z^E_{\mu^E}))$, the set of stable matchings for the regular admissions matching problem $Z^R(Z^E_{\mu^E}) \in \mathcal{Z}^R(Z^E_{\mu^E})$. In this set, there exists a matching $\mu^R_C(Z^R(Z^E_{\mu^E}))$, called the college-optimal stable matching in the regular admissions period, such that
$$\mu^R_C(Z^R(Z^E_{\mu^E}))(c)\, R_c\, \mu^R(c)$$
for all $c \in C$ and for all $\mu^R \in \mathcal{S}^R(Z^R(Z^E_{\mu^E}))$.

Analogously, there is a student-optimal stable matching in the regular admissions period, $\mu^R_S(Z^R(Z^E_{\mu^E}))$, that every student likes as well as any other

[12] We do not need to check the individual blocking of students unassigned to any college in early admissions since by definition $\mu^R(s)\, R_s\, \{\emptyset\}$ for all $s \in S$.

stable matching.[13]

Regular Matching Rules: A matching rule in the regular admissions period is a function $\varphi^R : \mathcal{Z}^R \to \mathcal{M}^R$ such that for all $Z^R \in \mathcal{Z}^R$, we have $\varphi^R(Z^R) \in \mathcal{M}^R(Z^R)$. Let $\bar{\varphi}^R$ denote the set of all such regular matching rules.

Stability of Regular Matching Rules: A regular matching rule $\varphi^R \in \bar{\varphi}^R$ is stable at an early matching rule $\varphi^E \in \bar{\varphi}^E$ if $\varphi^R(Z^R(Z^E_{\varphi^E(Z^E)})) \in \mathcal{S}^R(Z^R(Z^E_{\varphi^E(Z^E)}))$ for all $Z^E \in \mathcal{Z}^E$ and for all $Z^R(Z^E_{\varphi^E(Z^E)}) \in \mathcal{Z}^R(Z^E_{\varphi^E(Z^E)})$.

2.1.6 Matching Systems

For any $\varphi^E \in \bar{\varphi}^E$ that is used in the early admissions period and for any $\varphi^R \in \bar{\varphi}^R$ that is used in the regular admissions period, the ordered pair (φ^E, φ^R) is called a matching system. Let $\vec{\varphi}$ denote the matching system (φ^E, φ^R).

Stability of Matching Systems: A matching system $\vec{\varphi}$ is stable if (i) φ^E is stable, and (ii) φ^R is stable at φ^E.

Let $\vec{\varphi}_C$ be such that $\varphi^E_C(Z^E) = \mu^E_C(Z^E)$ and $\varphi^R_C(Z^R(Z^E_{\varphi^E_C(Z^E)})) = \mu^R_C(Z^R(Z^E_{\mu^E_C}))$, for all $Z^E \in \mathcal{Z}^E$ and $Z^R(Z^E_{\varphi^E_C}(Z^E)) \in \mathcal{Z}^R(Z^E_{\varphi^E_C(Z^E)})$. We call $\vec{\varphi}_C$ as the college-optimal stable matching system.

Similarly, let $\vec{\varphi}_S$ be such that $\varphi^E_S(Z^E) = \mu^E_S(Z^E)$ and $\varphi^R_S(Z^R(Z^E_{\varphi^E_S(Z^E)})) = \mu^R_S(Z^R(Z^E_{\mu^E_S}))$, for all $Z^E \in \mathcal{Z}^E$ and $Z^R(Z^E_{\varphi^E_S(Z^E)}) \in \mathcal{Z}^R(Z^E_{\varphi^E_S(Z^E)})$. We call $\vec{\varphi}_S$ as the student-optimal stable matching system.

2.2 Early Admissions Game

We consider a matching environment $F = (C, S, q, R)$ with $|C| \geq 2$ for the game to be nontrivial. We assume that the environment F and the matching system $\vec{\varphi}$ are common knowledge. We also assume that colleges com-

[13] To find the college-optimal and student-optimal stable matchings in the two admissions periods, we respectively use the well-known college-proposing and student-proposing deferred acceptance algorithms by Gale and Shapley (1962).

pletely know students' early lists $\Sigma_S^E(\alpha)$ for each possible announcement of the early admissions plans $\alpha \in \{a, d\}^m$ as well as the regular admissions market $Z^R(Z^E_{\varphi^E(Z^E)})$ for each possible realization of the early admissions market $Z^E \in \mathcal{Z}^E$. The game consists of three consecutive stages:

Stage 1: Colleges simultaneously choose one of the two early admissions plans, namely early action and early decision, to be implemented in the second stage of the game. Once a profile $\alpha \in \{a, d\}^m$ is determined, it becomes common knowledge in the rest of the game.

Stage 2: An early admissions market opens at the beginning of the second stage of the game. Observing the profile α and the corresponding vector $\Sigma_S^E(\alpha)$, each college c simultaneously chooses its early list $\Sigma_c^E(\alpha)$, its early quota $q_c^E(\alpha)$, and its pre-matching partition $\Pi_c^E(\alpha)$. Given the early admissions market $Z^E(\alpha) = (\alpha, q_C^E(\alpha), \Sigma_C^E(\alpha), \Sigma_S^E(\alpha), \Pi_C^E(\alpha))$, the early matching rule φ^E specifies for each college the list of students to accept early within its early admissions quota. Consequently, the post-matching summary $Z^E(\alpha)_{\varphi^E(Z^E(\alpha))}$ of the early admissions market becomes common knowledge.

Stage 3: The regular admissions market $Z^R(Z^E(\alpha)_{\varphi^E(Z^E(\alpha))})$ opens at the beginning of the third stage of the game. Given the commitments of colleges in the early admissons period, vacant slots of colleges are filled according to the matching rule φ^R.

Above, we have described a multiple stage game with observable actions. Clearly, the third stage of the game involves no strategies to be played. Thus, in order to solve for a subgame perfect equilibrium of the game, we proceed from stage 2 backwards.

For a given profile α and a matching system $\vec{\varphi}$, college c's preferences in stage 2 ($S2$) over the reports of early lists, early admissions quotas and pre-matching actions of colleges are represented by a binary relationship $\succeq_c^{\vec{\varphi},S2}$ over the set $\mathcal{Z}^E_{\alpha,C}$ such that for all $Z, Z' \in \mathcal{Z}^E_{\alpha,C}$ we have

$Z \succeq_c^{\vec{\varphi},S2} Z'$ if and only if $\varphi^R(Z^R(\Gamma_\alpha(Z)_{\varphi^E(\Gamma_\alpha(Z))}))\ R_c\ \varphi^R(Z^R(\Gamma_\alpha(Z')_{\varphi^E(\Gamma_\alpha(Z'))}))$.

Define college c's best response correspondence in stage 2 under the matching system $\vec{\varphi}$ by $\beta_c^{\vec{\varphi},S2} : \mathcal{Z}^E_{\alpha,-c} \to \mathcal{Z}^E_{\alpha,c}$ such that for any $Z_{-c} \in \mathcal{Z}^E_{\alpha,-c}$, we have

$$\beta_c^{\vec{\varphi},S2}(Z_{-c}) = \{Z'_c \in \mathcal{Z}^E_{\alpha,c} : (Z'_c, Z_{-c}) \succeq_c^{\vec{\varphi},S2} (Z''_c, Z_{-c}) \text{ for all } Z''_c \in \mathcal{Z}^E_{\alpha,c}\}.$$

Given any $\alpha \in \{a,d\}^m$, a pure strategy (Nash) equilibrium of stage 2 game is a strategy profile $\hat{Z}(\alpha) \in \mathcal{Z}^E_{\alpha,C}$ such that $\hat{Z}_c(\alpha) \in \beta_c^{\vec{\varphi},S2}(\hat{Z}_{-c}(\alpha))$ for all $c \in C$.

Assume that for each $\alpha \in \{a,d\}^m$, the set of equilibrium strategy profiles of stage 2 game is always nonempty and each member of this set leads to the same matching outcome $\varphi^E(\Gamma_\alpha(\hat{Z}(\alpha)))$.[14] Then going backwards to stage 1 of the reduced game, we represent college c's preferences in stage 1 ($S1$) over the early admissions plans by a binary relationship $\succeq_c^{\vec{\varphi},S1}$ over $\{a,d\}^m$ such that for all $\alpha', \alpha'' \in \{a,d\}^m$ we have $\alpha' \succeq_c^{\vec{\varphi},S1} \alpha''$ if and only if

$$\varphi^R(Z^R(\Gamma_{\alpha'}(\hat{Z}(\alpha'))_{\varphi^E(\Gamma_{\alpha'}(\hat{Z}(\alpha')))}))\ R_c\ \varphi^R(Z^R(\Gamma_{\alpha''}(\hat{Z}(\alpha''))_{\varphi^E(\Gamma_{\alpha''}(\hat{Z}(\alpha'')))})).$$

Define college c's best response correspondence in stage 1 under the matching system $\vec{\varphi}$ by $\beta_c^{\vec{\varphi},S1} : \{a,d\}^{m-1} \to \{a,d\}$ such that for any $\alpha_{-c} \in \{a,d\}^{m-1}$, we have

$$\beta_c^{\vec{\varphi},S1}(\alpha_{-c}) = \{\alpha'_c \in \{a,d\} : (\alpha'_c, \alpha_{-c}) \succeq_c^{\vec{\varphi},S1} (\alpha''_c, \alpha_{-c}) \text{ for all } \alpha''_c \in \{a,d\}\}.$$

Given the Nash play in stage 2 game, a pure strategy (Nash) equilibrium of stage 1 game is a strategy profile $\alpha \in \{a,d\}^m$ such that $\alpha_c \in \beta_c^{\vec{\varphi},S1}(\alpha_{-c})$ for all $c \in C$.

Apparently, the list $(\hat{\alpha}, \hat{q}^E_C(\hat{\alpha}), \hat{\Sigma}^E_C(\hat{\alpha}), \hat{\Pi}^E_C(\hat{\alpha}))$ constitutes a pure strategy subgame perfect equilibrium of the two-period college admissions game, if the list $(\hat{q}^E_C(\hat{\alpha}), \hat{\Sigma}^E_C(\hat{\alpha}), \hat{\Pi}^E_C(\hat{\alpha}))$ is a pure strategy Nash equilibrium of stage 2 game and the profile $\hat{\alpha}$ is a pure strategy Nash equilibrium of the reduced game in stage 1.

We now introduce the following definition that will be useful in the next section while characterizing the equilibria of the described game.

For any finite set X, any linear order $\tilde{R}$ defined over X, and any positive integer $l \leq |X|$, denote by $Top(X;l)$ the top lth-ranked element of X under $\tilde{R}$.

[14] We will check that this is indeed the case in the *essentially* unique equilibrium of the game.

3 Results

We will below show that given any profile of early admissions plans, it is a weakly dominant strategy for each college to report in the second stage of the described game its early quota as its total capacity, its early preference list as the restriction of its regular preference ordering on any collection of acceptable students involving the top acceptable students within its capacity size, the set of rejected students consisting of all students that are not acceptable with respect to its regular preference ordering, the set of outright deferred students to be consisting of all the nonrejected student applicants ranking outside its capacity size with respect to its regular preference ordering, and the set of early admissible students as simply the rest of the early applicants.

Theorem 1. *For any $\alpha \in \{a, d\}^m$, the list $q_c^E(\alpha) = q_c$, $\Sigma_c^E(\alpha) \supseteq \{s \in S : Top(\Sigma_c; \rho) = s$ for some $\rho \in \mathcal{Q}_c^E(q_c)$ and $\{s\} \, P_s \, \emptyset\}$, $J_c(\alpha) = \{s \in \Phi_c^E(\alpha) : \emptyset \, P_c \, \{s\}\}$, $D_c(\alpha) = \{s \in \Phi_c^E(\alpha) \backslash J_c(\alpha) : s = Top(\Sigma_c^E(\alpha), \rho)$ for $\rho > q_c\}$, and $A_c(\alpha) = \Phi_c^E(\alpha) \backslash (D_c^E(\alpha) \cup J_c^E(\alpha))$ constitutes a weakly dominant strategy for each college c in stage 2 of the college admissions game.*

Proof. Consider any $\alpha \in \{a, d\}^m$, $q^E(\alpha) \in \mathcal{Q}^E(q)$, $\Sigma_c^E(\alpha) \subset \{T \in \Sigma_c : T \, P_c \, \emptyset\}$ for all $c \in C$, $J_c(\alpha) \subseteq \Phi_c(\alpha)$ for all $c \in C$. We will first show that $D_c(\alpha) = \{s \in \Phi_c(\alpha) \backslash J_c(\alpha) : s = Top(\Sigma_c^E(\alpha), \rho)$ for $\rho > q_c\}$ is weakly dominant strategy for all $c \in C$ such that $\alpha_c = a$. The reason is that by deferring a student s instead of accepting him or her early, a college c can get rid of its unilateral commitment, hence from the risk of filling early a slot in its total capacity with a student that may be inferior to a regular applicant. However, if student s is in the list of the most preferred q_c students of college c, deferring and accepting are apparently equivalent in terms of the matching outcome they would induce. (Note that by precautiousness of students in regular admissions, the set of colleges that are preferred by s to college c in the regular admissions period are the same, so that by deferring student s college c cannot become worse off than in the case it accepts s early.) The same conclusions also carry over to the case in which $\alpha_c = d$, since a student can apply to only one college offering early decision plan. Then, it follows that $J_c(\alpha) = \{s \in \Phi_c(\alpha) : \emptyset\, P_c\, \{s\}\}$ is a weakly dominant strategy for college $c \in C$ for all $\alpha \in \{a, d\}^m$. $A_c(\alpha)$ follows by definition. Then $\Sigma_c^E(\alpha) \supseteq \{s \in S : Top(\Sigma_c; \rho) = s$ for some $\rho \in \mathcal{Q}_c^E(q_c)$ and $\{s\}\, P_s\, \emptyset\}$ is a weakly dominant strategy for each college $c \in C$. ∎

Although the early preference list of any college in the characterized equilibrium of stage 2 game is not uniquely given, the equilibrium is *essentially* unique, since the set of early admissible students and hence the matching outcome for a given early matching rule are uniquely characterized. Thus, we can now move backwards to stage 1 and find the admissions plans of colleges in equilibrium. The below theorem shows that given the equilibrium strategies in stage 2, the choice of early action plan in stage 1 is a weakly dominant strategy for each college whenever each student applies to an early decision college only if it is the top college in his or her early list and weakly preferred to the top college in his or her regular list.

Theorem 2. *Consider the list* $(\alpha, Z_C(\alpha))$ *such that for all* c, $\alpha_c = a$, $Z_c(\alpha) = (q_c^E(\alpha), \Sigma_c^E(\alpha), \Pi_c(\alpha))$, *where* $q_c^E(\alpha) = q_c$, $\Sigma_c^E(\alpha) \supseteq \{s \in S : Top(\Sigma_c; \rho) = s$ *for some* $\rho \in \mathcal{Q}_c^E(q_c)$ *and* $\{s\}\, P_s\, \emptyset\}$, *and* $\Pi_c(\alpha) = \{J_c(\alpha), D_c(\alpha), A_c(\alpha)\}$ *with* $J_c(\alpha) = \{s \in \Phi_c^E(\alpha) : \emptyset\, P_c\, \{s\}\}$, $D_c(\alpha) = \{s \in \Phi_c^E(\alpha) \backslash J_c(\alpha) : s = Top(\Sigma_c^E(\alpha), \rho)$ *for* $\rho > q_c\}$, *and* $A_c(\alpha) = \Phi_c^E(\alpha) \backslash\, (D_c^E(\alpha) \cup J_c^E(\alpha))$. *The list* $(\alpha_c, Z_c(\alpha))$ *constitutes a weakly dominant strategy for each college* c *in the college admissions game if for any* $s \in S$ *there exists a college* $c' \in \Sigma_s^E(\alpha)$ *such that* $\alpha_{c'} = d$ *only if* $c' = Top(\Sigma_s^E(\alpha); 1)$ *and* $Top(\Sigma_s^E(\alpha); 1)\, R_s\, Top(\Sigma_s^R(x_S^E(\Gamma_\alpha(Z_C(\alpha))_{\varphi^E(\Gamma_\alpha(Z_C(\alpha)))})); 1)$.

Proof. In the proof of Theorem 1, we have shown that $q_c^E(\alpha) = q_c$, $\Sigma_c^E(\alpha) \supseteq \{s \in S : Top(\Sigma_c; \rho) = s$ for some $\rho \in \mathcal{Q}_c^E(q_c)$ and $\{s\}\, P_s\, \emptyset\}$, $J_c(\alpha) = \{s \in \Phi_c^E(\alpha) : \emptyset\, P_c\, \{s\}\}$, $D_c(\alpha) = \{s \in \Phi_c^E(\alpha) \backslash J_c(\alpha) : s = Top(\Sigma_c^E(\alpha), \rho)$ for $\rho > q_c\}$, and $A_c(\alpha) = \Phi_c^E(\alpha) \backslash\, (D_c^E(\alpha) \cup J_c^E(\alpha))$ constitute for each college c a weakly dominant strategy in stage 2 of the college admissions game. Define $Z_c(\alpha) = (q_c^E(\alpha), \Sigma_c^E(\alpha), \Pi_c(\alpha))$, where $\Pi_c(\alpha) = \{J_c(\alpha), D_c(\alpha), A_c(\alpha)\}$. Given the associated early admissions market $\Gamma_\alpha(Z_C(\alpha))$, the strategy $\alpha_c = a$ weakly dominates the strategy $\alpha_c = d$ for each college c. This is because of the assumption that for all $c \in C$, for all $\alpha_{-c} \in \{a, d\}^{m-1}$, and for all $s \in S$, $c \in \Sigma_s^E((d, \alpha_{-c}))$ implies $c \in \Sigma_s^E((a, \alpha_{-c}))$. Hence, if $s \in \varphi^E(\Gamma_\alpha(Z_C(d, \alpha_{-c})))$, then $s \in \varphi^E(\Gamma_\alpha(Z_C(a, \alpha_{-c})))$. On the other hand, for all $c \in C$, for all $\alpha_{-c} \in \{a, d\}^{m-1}$, and for all $s \in S$ if $c \notin \Sigma_s^E((d, \alpha_{-c})$ then either there exists $c' \in \Sigma_s^E((d, \alpha_{-c}))$ such that $\alpha_{c'} = d$ or the student s is not applying to any college offering an early decision plan. In both cases by setting $\alpha_c = a$ college c is never worse off; indeed it may even entice the student s to apply early to itself in case he or she also applies to a higher ranked college with an early decision plan. ∎

The (weak) superiority of early action plan for each college to early decision plan is in line with the earlier result of Mumcu and Saglam (2007) in a simpler two-period admissions model with early decision showing that it is a weakly dominant strategy for each college to terminate its early decision program if every student, choosing to act early, always applies early to his or her top choice college. However, we also establish in this study the (weak) superiority of two-period admissions with early action to single-period (regular) admissions.

We should here notice that early action and early decision programs can coexist in equilibrium if we relax the assumption that each student applies to an early decision college only if it is the top college in his or her early list and weakly preferred to the top college in his or her regular list. For example, consider a matching environment involving a very popular early action college, c_1, in regular admissions that however receives early applications from moderately ranked students, a relatively less popular early decision college, c_2, in regular admissions that receives early applications of very high ranked students, and the least popular college, c_3, in regular admissions that receives early applications from high, medium, and low ranked students. One can easily fill in the remaining details of the matching environment to ensure that the given early admissions plans are in equilibrium since c_1 prefers early action to early decision not to tie itself to some moderate students in early admissions, college c_2 oppositely prefers early decision to early action in order to secure for itself early admittances of some compromising, high ranked students, and finally college c_3, observing the strategy of college c_2, prefers early action to early decision not to lose early applications from some compromising, high and medium ranked students.

4 Conclusions

We have studied some strategic aspects of the early admissions problem faced by colleges in the United States using a two-period matching game with observable actions. Our first result shows that irrespective from the early admissions plans in the matching market, it is a weakly dominant strategy for each college to choose (i) its early quota as its total capacity, (ii) its early preference list as any collection of acceptable students involving the top acceptable students within its capacity, (iii) the set of rejected students to be consisting of all unacceptable students, (iv) the set of outright deferred students to be consisting of all the nonrejected applicants ranking outside its capacity size, (v) the set of early admissible students as the rest of the early applicants, and (vi) the set of ex-post deferrals (by assumption) as the union of the set of outright deferred students and the set of all early admissible students with whom it was not matched in the early admissions period.

The validity of our 'quite inclusive' equilibrium strategy of deferral, which we also find to be independent of the profile of early admissions plans, is verified by the report of Avery et.al. (2003, pp. 188-189) that "...historically most colleges rejected 5 percent or fewer of their early applicants in December. Some, such as Cornell, Georgetown, MIT, and Tufts, have automatically deferred to the regular pool all early applicants who are not admitted in December".

Our second result states that for each college early action plan is weakly dominating early decision plan whenever each student in the market applies to an early decision college only if it is the top college in his or her early list and weakly preferred to the top college in his or her regular list. The early application strategy driving our second result is strongly recommended for all students by the College Board, counsellars, college admission officers, and many college guides. Indeed, this recommendation might have been highly welcome by students and their families, as a survey reported in Avery et.al (2003, p. 205) reveals that 98 percent of a total of 48 sample students applied during 1997-2000 to an early decision program as a strong or weak first choice. It is quite interesting that a common recommendation solely made for the better functioning of early decision plans can in effect lead to the complete elimination of these plans in equilibrium, as also remarked by Mumcu and Saglam (2007).

Looking from an entirely different angle, Avery et.al. (2003, pp. 267-268) highlights a major concern, for students and their families, about early admissions pertaining only to early decision as follows:

> "Because Early Decision is binding, it prevents applicants from gathering more information during their senior year, and possibly changing their minds. It also prevents financial aid applicants from seeing, much less bargaining over, aid packages at other colleges. ... The simplest way to address these concerns would be to abolish Early Decision programs but allow Early Action to continue."

Our results obtained under the strategic behavior of colleges in the intertemporal allocation of their capacities may absolutely fortify the arguments for 'eliminating early decision and keeping early action' as one of possible reforms[15] to the existing college admissions system in a period of increasing abstention of colleges from early decision.[16]

However, as we have already remarked, the separate incentives of colleges in implementing early action and early decision plans can coexist in equilibrium if we depart from the premises of our second theorem by allowing the presence of risk averse students who compromise in early admissions.

Thus, we now have a better understanding that the correct choice as well as the success of a reform to the college admissions system can only be accomplished by gathering information about the genuine preferences of students and colleges and giving due consideration to their strategic incentives.

[15] Avery et.al. (2003, pp. 266-293) discusses in detail seven reforms to the existing college early admissions system.

[16] Avery et.al. (2003, p. 272) remarks that "as of fall 2002, the efforts of Richard Levin, the president of Yale, have helped to induce four colleges to switch from Early Decision to Early Action and led others to question the value of their Early Decision programs as well."

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